Lecture 22

11.2 - Series

In the last section, we talked about sequences... in this section, we add up those lists of numbers. Such a sum is called a series. Just like sequences, series can be infinite or finite. Given a sequence $\{a_n\}_{n=1}^{\infty}$, the series which is adding up its terms is denoted:

$$a_1 + a_2 + a_3 + \cdots = \sum_{j=1}^{\infty} a_j$$

Ex: $\sum_{j=1}^{\infty} \frac{1}{2^j} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$.

Def: Given a series $\sum_{j=1}^{\infty} a_j$, the n^{th} partial sum is

$$s_n =$$

If the sequence $\{s_n\}$ is convergent and $\lim_{n\to\infty} s_n = S$, then we say the series $\sum_{j=1}^{\infty} a_j$ is convergent, and we let

$$\boxed{\sum_{j=1}^{\infty} a_j = \lim_{n\to\infty} \sum_{j=1}^{n} a_j = \lim_{n\to\infty} s_n = S}$$

S is called the sum of the series. Otherwise, we call the series divergent.

Ex: Find a formula for the n^{th} partial sum of the series $\sum_{j=1}^{\infty} \frac{1}{2^j}$. Is this series convergent? If so, what is its sum?

Ex: Repeat the last example for the series $\sum_{j=1}^{\infty} j$.

Geometric Series

A geometric series is a series of the form

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \cdots$$

Theorem: The geometric series $\sum_{n=1}^{\infty} ar^{n-1}$ converges if

If , the series diverges.

proof:

Ex: Find the sum of the series:

$$\sum_{n=1}^{\infty} \frac{3(-1)^n}{5^n}$$

Ex: Express $0.777777\cdots = 0.\overline{7}$ as a fraction.

Telescoping Series

A **telescoping series** is one of the form:

Because of its form, its partial sums are pretty simple: $S_n =$

Ex: Determine the sum of:

$$\sum_{j=1}^{\infty} \frac{1}{j^2+3j+2}$$

Harmonic Series

The <u>harmonic series</u>, $\sum_{n=1}^{\infty} \frac{1}{n}$, is an example of a <u>divergent</u> series. This example is important for comparison tests and counter examples. We'll prove it diverges in the next section.

<u>Remark</u>: Adding or removing a <u>finite</u> # of terms from the beginning of a series will not affect its convergence (or divergence), e.g.,

$\sum_{j=-4}^{\infty} \frac{1}{2^j}$ is still convergent & $\sum_{j=1000}^{\infty} \frac{1}{j}$ is still divergent.

Divergence Test

⚠ The converse is NOT TRUE! That is:

Ex: Do the series:

$$\sum_{n=1}^{\infty} \frac{n^3}{2n^3-2} \quad \& \quad \sum_{n=1}^{\infty} \frac{n^2}{2^n}$$

diverge?

Properties of Series

Suppose $\sum a_n$ & $\sum b_n$ are convergent and c is a number. Then $\sum (a_n+b_n)$, $\sum (a_n-b_n)$, and $\sum ca_n$ all converge, and

$\sum (a_n+b_n) =$ $\qquad\qquad$ $\sum (a_n-b_n) =$

$\sum ca_n =$

Ex: Find the sum of

$$\sum_{n=1}^{\infty}\left(\frac{1+2^n}{3^n}\right)$$

Ex: For what values of x does $\sum_{n=1}^{\infty} \frac{2x^n}{7^{n-1}}$ converge?

Ex: Does $\sum_{n=10}^{\infty} \ln\left(\frac{n^2+1}{2n^2+1}\right)$ diverge?